Course Notes:
CNS 221
Computational Neuroscience
Lecture 1

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1 Nernst Equation

$$E = -zqV$$

Where :

- z is the valence of the ion
- q is the elementary charge ($q = +1.6 * 10^{-19} As$)

$$p(E > E_{th}) \quad = \quad e^{-(\frac{E}{k_B T})} \tag{1}$$

Nernst Equation

$$Vm = \frac{k_B T}{zq} \ln \frac{[Conc._{out}]}{[Conc._{in}]} \tag{2}$$

Where :

- $k_B = 1.38 * 10^{-23} \frac{m^2 kg}{s^2 K}$ is the Boltzman Constant
- T is temperature

2 Goldman Hodgkin Katz equation

$$V = \frac{k_B T}{zq} \ln \frac{P_k[K^+]_{out} + P_{Na}[Na^+]_{out} + P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in} + P_{Na}[Na^+]_{in} + P_{Cl}[Cl^-]_{out}} \tag{3}$$

3 Ohm's Law, Kirchhoff's Laws

3.1 Ohm's Law

$$V = IR \tag{4}$$

3.2 kirchoffs 1st law

All current that enters a node will exit the node so therefore the net current at any given node is zero.

$$\sum_i I_i = 0 \tag{5}$$

3.3 kirchoff's second law: aka the loop rule

If you move around a loop in a circuit, the summed Voltage should equal zero.
For a nice visualization see:
`http://regentsprep.org/Regents/physics/phys03/bkirchof2/default.htm`

4 the circuit model of the cell membrane

4.1 Applying Ohm's and Kirchoff's Laws to the Cell membrane

The currents across the membrane can be described as

$$I_{memb} = I_C + I_K + I_{Na} + I_{Cl} \tag{6}$$

$$I_{memb} = C\frac{dV}{dt} + \frac{V_m - E_K}{R_K} + \frac{V_m - E_{Na}}{R_{Na}} + \frac{V_m - E_{Cl}}{R_{Cl}} \tag{7}$$

$$I_{memb} = C\frac{dV}{dt} + g_K(V_m - E_K) + g_{Na}(V_m - E_{Na}) + g_{Cl}(V_m - E_{Cl}) \tag{8}$$

at rest $I_{memb} = 0$ and $\frac{dV}{dt} = 0$ so the equation becomes

$$0 = g_K(V_m - E_K) + g_{Na}(V_m - E_{Na}) + g_{Cl}(V_m - E_{Cl}) \tag{9}$$

$$0 = g_K V_m - g_k E_k + ... \tag{10}$$

$$g_K E_K + g_{Na}E_{Na} + g_{Cl}E_{Cl} = g_K Vm + g_{Na}Vm + g_{Cl}Vm \tag{11}$$

$$g_K E_K + g_{Na}E_{Na} + g_{Cl}E_{Cl} = V_m(g_K + g_{Na} + g_{Cl}) \tag{12}$$

$$\frac{g_K E_K + g_{Na}E_{Na} + g_{Cl}E_{Cl}}{(g_K + g_{Na} + g_{Cl})} = V_m \tag{13}$$

4.2 Injecting a test Current

fyi

$$Q = CV \tag{14}$$

$$\frac{dQ}{dt} = \frac{dC}{dt}\frac{dV}{dt} \tag{15}$$

$$I = C\frac{dV}{dt} \tag{16}$$

Following the derivation in Biophysics of Computation pp 9-10

$$C\dot{V} = g(V_m - V_r) + I_{inj} \tag{17}$$

$$RC\dot{V} = (V_m - V_r) + RI_{inj} \tag{18}$$

$$\tau\dot{V} = (V_m - V_r) + RI_{inj} \tag{19}$$

From 1st term ODEs we should recognize that equation (19) is a first order differential equation with the general solution of:

$$Vm(t) = v_0 e^{-(t/\tau)} + v_1 \tag{20}$$

At $t = \infty$ the

$$v_0 e^{-(t/\tau)} \tag{21}$$

will go to zero. Therefore

$$Vm(\infty) = v_1 \tag{22}$$

and for a constant current injection this will result in

$$v_1 = V_r + RI \tag{23}$$

Furthermore at

$$V_m(0) = V_r \tag{24}$$

$$v_0 + v_1 = V_r \tag{25}$$

$$v_0 + V_r + RI = V_r \tag{26}$$

$$v_0 = -RI \tag{27}$$

So plugging this all back in...

$$V_m(t) = -RIe^{-(t/\tau)} + V_r + RI \tag{28}$$

$$V_m(t) = RI(1 - e^{-(t/\tau)}) + V_r \tag{29}$$

And τ the time constant of this system is equal to

$$\tau = RC \tag{30}$$

5 Linear System

For a review check-out:
`http://www.cns.nyu.edu/~david/linear-systems/linear-systems.html`

$$V(t) = L[I_{inj}(t)] \tag{31}$$

$$I(t) = \int_{-\infty}^{\infty} I(t)\delta(t-t')dt' \tag{32}$$

Remember

$$\delta(t) = 0 \quad \forall \quad t \neq 0 \tag{33}$$

$$\delta(0) = +\infty \tag{34}$$

$$\int_{-\infty}^{\infty} \delta(t) = 1 \tag{35}$$

$$V(t) = (I * h)(t) \tag{36}$$

$$= \int_{-\infty}^{\infty} I(t') \times h(t-t')dt' \tag{37}$$

where

$$h = \mathbf{L}[\delta(t)] \tag{38}$$

By virtue of this being a linear system we know:

$$\mathbf{L}[a+b] = \mathbf{L}[a] + \mathbf{L}[b] \tag{39}$$

Stating with the initial equation for the membrane we will take the fourier transform.

$$\tau\dot{V}(x) = -V(x) + RI_{inj}(t) \tag{40}$$

$$i\omega\tilde{V}(w)\tau = -\tilde{V}(w) + RI_{inj}(t) \tag{41}$$

$$\tilde{V}(\omega) = \frac{R}{1+i\omega\tau} \tag{42}$$

We take the fourier transform in this case because a convolution as seen in equation 37 is equivalent to the following set of operations:

1. Taking the fourier or laplace transform of each equation

2. Multiply the transformed equations together
3. Take the inverse transfor of the product
4. Revel in the ease of performing a convolution using this method.

For a good animation of a convolution see:
`http://mathworld.wolfram.com/Convolution.html`